AN ROINN OIDEACHAIS

LEAVING CERTIFICATE EXAMINATION, 1964.

APPLIED MATHEMATICS—HONOURS.

MONDAY, 15th JUNE—Morning, 10 to 12.30.

Not more than six questions may be answered. All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

1. A uniform ladder PQ has a length of 26 feet and weighs 40 lbs. The end Q leans against a vertical rough wall (coefficient of friction $\frac{1}{4}$) and the other end P is on rough horizontal ground (coefficient of friction $\frac{1}{2}$) at a distance of 24 feet from the wall. A block weighing 25 lbs. is placed on the ground against the bottom of the ladder so as to just prevent the ladder from slipping. Find the coefficient of friction between the block and the ground.

2. If the co-ordinates of the vertices of a triangle are (x_1, y_1), (x_2, y_2), (x_3, y_3) and if $(\bar{x}, \bar{y})$ are the co-ordinates of the centroid of the triangle, show that

$$\bar{x} = \tfrac{1}{3}(x_1 + x_2 + x_3) \text{ and } \bar{y} = \tfrac{1}{3}(y_1 + y_2 + y_3).$$

Hence, or otherwise, show that the centre of gravity of a triangular lamina is the same as that of three equal masses situated at the vertices of the triangle.

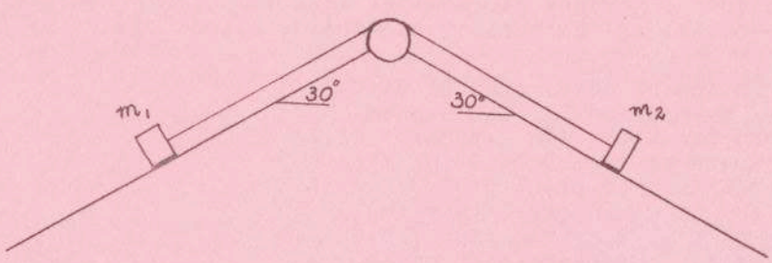
A lamina is in the shape of a trapezium ABCD in which AB and CD are parallel. AB = BC = 2 inches, CD = 4 inches and $\angle ABC = 60°$. Find the position of the centre of gravity of the lamina.

3. The direction of the wind relative to that of a bus travelling in the direction 60° South of East is from the East, while the direction of the wind relative to that of a car travelling in the direction 30° South of West is from the West. (The velocity of the wind is assumed constant). If the speed of the wind relative to that of the bus is added to the speed of the wind relative to that of the car, the total is 60 m.p.h. Find the speed of the bus and the speed of the car.

4. A train is pulled by its engine up an incline of 1 in 80 at a uniform speed of 25 m.p.h. against a resistance to motion of 17 lb. wt. per ton. If the weight of the train and its engine is 440 tons, find the horse-power at which the engine is working.

If the train travels on a level track at a speed of 18 m.p.h. against a resistance to motion of 17 lb. wt. per ton, find its acceleration if the engine is developing the same horse-power as before.

5. A system consists of two masses m_1, m_2 suspended by a light inextensible string over a light frictionless pulley. The pulley is at the intersection of two smooth planes equally inclined to the horizontal at an angle of 30°(see diagram) and the masses weigh 9 oz. and 7 oz., respectively. The system, initially held at rest, is released and after 2 sec. of motion a mass of weight 4 oz. is detached from the 9 oz. mass. Find the distance traversed by the 7 oz. mass before it again reaches its initial position.



6. Define Simple Harmonic Motion.

A particle is moving in a straight line with simple harmonic motion. When it is 2 feet from its mean position its velocity is 6 feet per second and its acceleration is 6 feet per second^2. Find the amplitude and the period of the motion.

How long does the particle take to travel a distance 2 feet from its mean position and how far from its mean position is the particle in $\frac{1}{8}$ of the period ?

7. A roof may be taken to be a smooth plane sloping at an angle of 30° to the horizontal with its lower edge along the top of a vertical wall. A body slides down the roof and hits the ground (assumed horizontal) at an angle whose tangent is $\frac{7\sqrt{3}}{9}$. When the speed of the body is 8 ft. per sec. it has a distance of 16 feet to traverse before it leaves the roof. Find the height of the wall.

8. A circular piece of tin rotates at the rate of 30 revolutions per minute about an axis through its centre O, the axis being perpendicular to the plane of the tin. A and B are two points on the tin such that OA = 1 foot, OB = 2 feet and $\angle AOB = 60°$. Weights of 4 lb. and 2 lb. are placed respectively at A and B. Find the magnitude and direction of the resultant centrifugal force on the axis.

9. A vessel in the shape of a cube has internal edges each 2 ft. long. A liquid, of specific gravity 1·25, fills the vessel to a certain height and water which does not mix with the liquid occupies the remainder of the vessel. On each vertical side the total thrust on that part in contact with the water is equal to the total thrust on that part in contact with the liquid. Calculate the depth of the liquid of specific gravity 1·25 in the vessel.