LEAVING CERTIFICATE EXAMINATION, 1963.

APPLIED MATHEMATICS—HONOURS.

WEDNESDAY, 19th JUNE—Morning, 10 to 12.30.

Not more than six questions may be answered. All questions are of equal value.
Mathematical Tables may be obtained from the Superintendent.

1. P and S are two fixed pegs in the same horizontal line. One end of a light string is attached to P and the other end to S. When two masses are attached to the string, one at a point Q and the other at a point R, the angles PQR, QRS, RSP are 135° , 105°, 45°, respectively. Find the weight of each mass if the two together weigh 9 lbs.

2. ABCD is a quadrilateral lamina in which AB = 4 cms., BC = 12 cms., CD = 13 cms., DA = 3 cms. and $\angle BAD = 90°$. Find the perpendicular distance of the centre of gravity of ABCD from BD.

Find, also, the perpendicular distance of the centre of gravity of ABC from AD.

3. To a person on a ship travelling due East at 25 m.p.h. another ship 5 miles due North appears to be travelling at 15 m.p.h. in a direction 30° West of South. Find the velocity of the second ship in magnitude and direction as accurately as you can by using your tables.

Find the distance between the ships when they are nearest to one another.

4. An engine develops a horse-power of 272 as it accelerates at the rate of $\frac{1}{2}$ ft. per sec^2. when travelling at a speed of 30 m.p.h. down an incline of 1 in 80. If the weight of the engine is 200 tons, calculate the frictional resistance to motion in lb. wt. per ton.

5. A pile of mass one ton is driven vertically $1\frac{1}{2}$ feet into the ground by 10 blows of a hammer of mass 2 tons which falls vertically through a height of 9 feet.

Hence find the least mass which, when placed on top of the pile, would begin to drive the pile down. (Assume that the resistance of the ground is uniform and neglect the resistance-effects of side-friction on the pile.)

6. A mass of 20 ounces attached to a fixed point by a light inextensible string of length 3 feet describes a horizontal circle at a uniform rate of 420 radians per minute. Find the radius of the circle in feet and the tension in the string in lb. wt., correct to two significant figures in each case.

What angle would the string make with the vertical if the angular velocity were increased to 840 radians per minute ?

7. A dense fog is assumed to cover a horizontal plane to a uniform height of 224 feet. A mass projected from the plane rises above the fog after 2 sec. and then travels a horizontal distance of 800 feet before re-entering the fog. Find the initial velocity of the mass and its angle of projection.

8. A particle moves in a straight line so that its displacement x(cm.) from a fixed point at time t(sec.) is given by the formula

$$x = 3\sin 2t + 4\cos 2t.$$

Show that the motion is simple harmonic and find (i) the amplitude, (ii) the maximum velocity of the particle.

Find, also, the least time, correct to three significant figures, taken by the particle to travel a distance 2 cm. from its mean position.

9. PQR is a triangular lamina in which QR = 10", PR = 6" and PT = 5", where T is the mid-point of QR. The lamina is immersed in a vertical position in a liquid of specific gravity 0·84 so that the base QR is at the surface. Find the total thrust of the liquid on PQR.

How far must the lamina be withdrawn vertically upwards out of the liquid, the base QR remaining parallel to the surface, if the thrust on the portion in the liquid is $\frac{1}{2}$ lb. wt. ?

(One cubic foot of water weighs 62·5 lb.)