AN ROINN OIDEACHAIS

(Department of Education).

BRAINSE AN MHEAN-OIDEACHAIS

(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1957.

APPLIED MATHEMATICS.—Honours.

THURSDAY, 13th JUNE.—AFTERNOON, 2.30 TO 5.

Not more than *six* questions may be answered. All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

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1. If three co-planar non-parallel forces acting on a rigid body are in equilibrium, prove that their lines of action are concurrent and that the forces may be represented in magnitude and direction by the sides of a triangle taken in order.

A uniform bar AB which weighs 8 lb. is supported by two strings AC, BC attached to a fixed peg at C. If AB=6″, AC=5″, BC=3″, find the tension in each of the strings.

2. State the main laws of friction. Explain the term "coefficient of friction," and describe briefly how its value for two given surfaces may be found by experiment.

A box, lying on the floor of a railway carriage, just begins to slide back along the floor as the carriage is descending an incline of 1 in 8 with a uniform acceleration of 12 ft. per sec.^{2}. Show by diagram the forces acting on the box, and find the coefficient of friction between the box and the floor.

3. Show that the moment of a couple is the same about all points in the plane of the couple. Explain how couples may be compounded, and show that a force and a couple acting in the same plane are equivalent to a single force.

Prove that four forces represented in magnitude, direction *and position* by the sides of a quadrilateral taken in order are equivalent to a couple.

4. (i) A block is sliding freely down the line of greatest slope of a smooth inclined plane. If the velocity of the block increases from 3 ft. per sec. to 5 ft. per sec. in a distance of 5 feet, find the slope of the plane.

(ii) A car which weighs 1 ton is descending an incline of 1 in 80; the velocity of the car is 20 m.p.h. and it is accelerating at the rate of 2 ft. per sec.^{2}. If the frictional resistances to motion are equivalent to 55 lb. wt., find the horse-power at which the car is working.

5. A bullet weighing ·02 lb. is fired horizontally with a velocity of 2,000 ft. per sec. from a gun which is free to recoil. If the gun weighs 20 lb., find the velocity with which the gun begins to recoil, and find the total kinetic energy, in foot-lbs., of the gun and the bullet.

If the same bullet were fired from a gun weighing 5 lb., and the total kinetic energy was the same as above, show that the velocity of the bullet would be less by about 3 ft. per sec.

6. A particle is describing a circle of radius r, with constant angular velocity w: show that its acceleration is w^2r directed towards the centre of the circle.

A 2-ounce mass supported from a fixed point by a light inextensible string 4 feet long is describing a horizontal circle at a uniform rate of 2 revolutions per second. Find the tension in the string in lbs. wt., and find the vertical distance from the fixed point to the plane of the circle.

7. P, Q are two points at ground level 3,500 feet apart. An aeroplane is flying at a steady height of 1,600 feet above the ground in the direction QP, with a uniform velocity of 400 ft. per sec. When the aeroplane is directly over Q a shell is fired, at an angle of projection α, from a gun at P and strikes the aeroplane t seconds later. If $\cos\alpha=\frac{3}{5}$, find the value of t and the initial velocity of the shell.

8. A point A is describing a circle, centre O, at constant speed. If N is the foot of the perpendicular from A to a fixed diameter, show that N is moving with an acceleration which is directly proportional to its distance from O.

When N is 4 feet from O its velocity is 6 ft. per sec. away from O, and its acceleration is 16 ft. per sec.^{2} towards O. Find the amplitude and the period. How many seconds later does N reach O from that position? (Give your answer correct to the nearest tenth of a second.)

9. If a plane lamina is immersed vertically in a liquid at rest, prove that the total thrust on it due to the liquid is equal to the area of the immersed surface multiplied by the pressure at its centre of gravity.

A quadrilateral lamina ABCD in which BC=CD=5″ and AB=BD=AD=6″ is immersed in water, with the vertex A at the surface and the vertex C vertically below A. Find the total thrust of the water on the lamina in lbs. wt. correct to one decimal place.

[A cubic foot of water weighs 62·5 lb.]