AN ROINN OIDEACHAIS

(Department of Education).

BRAINSE AN MHEÁN-OIDEACHAIS

(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1954.

APPLIED MATHEMATICS.—Honours.

FRIDAY, 18th JUNE.—AFTERNOON, 2.30 TO 4.30.

Not more than *six* questions may be answered. All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

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1. A and B are the ends of a uniform bar which is 8 inches in length and which weighs 5 lb. The bar is supported by two strings AC and BC attached to a fixed peg at C. If AC=5 inches and BC=4 inches, find the tensions in the strings.

What weight attached at B would maintain the bar in a horizontal position ?

2. A non-uniform beam rests with one end in contact with a rough horizontal plane (coefficient of friction $\frac{\sqrt{3}}{2}$) and the other end in contact with a rough vertical wall (coefficient of friction $\frac{1}{2}$), the inclination of the beam to the horizontal being 30°. If the beam is just about to slip, in a vertical plane, find the ratio in which the length of the beam is divided by the position of its centre of gravity.

3. A car starts from rest and gathers speed. The following table gives the distance travelled by the car from its starting point in the corresponding time :—

Time (in seconds)	5	10	15	20	25	30
Distance (in feet)	30	96	173	274	399	618

Draw a distance-time curve and use it to estimate the velocity of the car at the end of (i) 15 seconds, (ii) 20 seconds, (iii) 25 seconds.

Plot those three values on a velocity-time curve, and so find approximately the acceleration of the car at the end of 20 seconds.

4. A circular disc of radius r is rolling in a straight line on a fixed horizontal plane, the centre of the disc having a uniform velocity v. Assuming that no sliding takes place, find the angular velocity of the disc about its centre. [*Question 4 continued overleaf.*]

If a point P is initially at the highest point of the disc, find the linear velocity of P at time t in magnitude and direction—giving the inclination to the horizontal.

Show that at any instant there are two points on the rim of the disc whose linear velocities are both equal to v in magnitude.

5. Starting from rest at the foot of an incline of 1 in 100, a cyclist travels up the incline with uniform acceleration until he has reached a speed of 15 m.p.h. ; he then continues at a steady speed of 15 m.p.h., so that 30 seconds after starting from rest he has travelled 550 feet up the incline. Find his uniform acceleration

If the cyclist and the cycle together weigh 160 lb., and if the frictional resistances to motion are equivalent to 3 lb. wt., find the horse-power at which the cyclist was working when his speed was 10 m.p.h.

6. A mass of 3 ounces is projected vertically upwards with an initial velocity of 80 ft. per sec. and 3 seconds later a mass of 1 ounce is projected vertically upwards from the same point with an initial velocity of 112 ft. per sec. If the masses coalesce on colliding, in how many seconds after the collision will the combined mass return to the point of projection ?

If the masses were projected as stated, except that each mass, at its time of projection, is given also a component horizontal velocity, show that the masses would still collide provided that the horizontal velocities were in the same direction and were in the ratio 1 : 5.

7. A motor cyclist is travelling at 31 m.p.h. round a curve of 200 feet radius on a track which is inclined to the horizontal at an angle of 10° (the plane of the curve being horizontal). Find the angle which the cyclist makes with the normal to the track.

If he can travel round the curve at 45 m.p.h. without skidding, find the least possible value of the coefficient of friction between the track and the tyres.

8. Define Simple Harmonic Motion.

The tension in a spiral spring is directly proportional to the distance through which it is extended beyond its natural length. When a mass m is suspended from it, the spring is extended a distance a beyond its natural length. If the mass is then pulled downwards from its equilibrium position and released, show that its subsequent motion is Simple Harmonic, and find the periodic time in terms of a.

9. Prove that the pressure is the same at all points at the same depth in a liquid at rest.

A solid cylinder is 4 inches in height and has a diameter of 3 inches, and its specific gravity is 2·4. The cylinder is suspended by a vertical string so that it is totally immersed in a liquid of specific gravity 0·8. The axis of the cylinder is vertical and the top of the cylinder is 2 inches below the surface of the liquid. Find the total thrust of the liquid (i) on the base of the cylinder, (ii) on the top of the cylinder ; and find also the tension in the string.