

# AN ROINN OIDEACHAIS

(Department of Education).

## BRAINSE AN MHEÁN-OIDEACHAIS

(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1953.

### APPLIED MATHEMATICS.—Honours.

WEDNESDAY, 17th JUNE.—MORNING, 10 TO 12.

Not more than *six* questions may be answered. All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

1. The ends of a light inextensible string 5 feet long are attached to two pegs which are 4 feet apart and in the same horizontal line. A mass of 20 lb. is suspended from the string at a point which is 2 feet from one peg and 3 feet from the other. Find the tension in each part of the string.

2. A block weighing 10 lb. can just rest without slipping on a rough plane when the plane is inclined at an angle of  $40^\circ$  to the horizontal. Find

(a) the magnitude of the least force in the horizontal direction,

(b) the magnitude and direction of the least force,

that will keep the block from slipping when the plane is inclined at an angle of  $50^\circ$  to the horizontal.

3. An engine weighing 50 tons is pulling a train of weight 150 tons down an incline of 1 in 80. The speed of the train is 30 m.p.h. and it is accelerating at the rate of  $\frac{1}{2}$  ft. per sec.<sup>2</sup> If the frictional resistances to motion are equivalent to 10 lb. wt. per ton, find the horse-power at which the engine is working.

4. A pile driver of mass 12 cwt. falls freely from rest through a distance of 9 feet and strikes a pile of mass 4 cwt. The pile driver and the pile move together as a single body after the impact and the pile is driven 6 inches into the ground. Find the average resistance of the ground, in tons weight.

5. What is meant by the velocity of one body relative to another ?

Explain, with the aid of a diagram, how the velocity of A relative to B may be found if the velocities of A and B are known.

To a person on a ship travelling due East at 20 m.p.h., another ship two miles due South appears to be travelling at 8 m.p.h. in a direction  $30^\circ$  West of North. Find the velocity of the second ship in magnitude and direction.

What is the distance between the ships when they are nearest to each other ?

6. If  $h$  is the greatest height reached by a projectile, prove that

$$h = \frac{u^2 \sin^2 a}{2g} = \frac{1}{8}gt^2$$

where  $u$  is the initial velocity,  $a$  the angle of projection, and  $t$  the total time of flight.

Two vertical posts, each of height  $l$  feet, stand  $\frac{1}{2}l$  feet apart on a horizontal plane. A particle is projected from a point on the plane at an angle of  $\tan^{-1} 3$  with the horizontal, and just clears the top of each post. If  $H$  is the greatest height reached by the particle, and  $v$  its initial velocity, show that

$$H = \frac{9v^2}{20g} = l + \frac{5gl^2}{16v^2}$$

and hence show that  $v^2 = \frac{5}{2}gl$ .

7. A mass of 10 ounces attached to a fixed point by a light inextensible string of length 2 feet is describing a horizontal circle at the uniform rate of 60 revolutions per minute. Find the radius of the circle in feet and the tension in the string in lb. wt., correct to one place of decimals in each case.

By how much is the tension increased if the speed is increased to 80 revolutions per minute?

8. Define Simple Harmonic Motion.

A particle moves in a straight line so that its displacement,  $x$ , from a fixed point at any time,  $t$ , is given by the formula

$$x = a \sin \omega t - b \cos \omega t.$$

Show that the motion is Simple Harmonic, and find an expression for (i) the amplitude, (ii) the maximum velocity.

Find, also, an expression for the least time taken by the particle to travel a distance  $d$  from its mean position.

9. A triangular lamina ABC is immersed in a vertical position in water with its vertex A at the surface and its base BC parallel to the surface. The base is 5 inches in length, and the height of the triangle is 3 inches. Find the total thrust of the water on ABC.

X, Y are two points on AB, AC respectively such that the straight line XY is horizontal. If the thrust on AXY is one-eighth of the thrust on ABC, find the depth of XY below the surface.

[One cubic foot of water weighs 62.5 lb.]