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(Department of Education).

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(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1944.

APPLIED MATHEMATICS—Honours.

THURSDAY, 22nd JUNE.—AFTERNOON 4 TO 6.

Not more than *six* questions may be answered. All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

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1. ABC is a steel rod, the portion AB being straight and of length 4 feet and the portion BC being curved. The whole rod lies in one plane. The end A is hinged to the floor and a load of 2 cwt. is suspended from C by a rope which is 3 feet from AB which is kept vertical by applying a horizontal force $3\frac{1}{2}$ feet from A. The weight of the whole rod may be taken to be $\frac{1}{2}$ cwt. acting at a distance of one foot from AB when the latter is in the vertical position. Find the horizontal force and the reaction at the hinge in magnitude and direction.

2. ABCD is a trapezium, AB being parallel to DC. AB $= a$, DC $= b$. Show that the centroid of the area is on the line joining the mid-points E, F of AB, DC and that it divides EF in the ratio $(a + 2b) : (2a + b)$.

3. It is found that when a plane is inclined at 30° to the horizontal a block of iron weighing 8 lb. slides uniformly down the plane. Show that the angle of friction is 30°.

When the plane is inclined at 45° to the horizontal the block is dragged slowly up the plane by a cord inclined at an angle θ to the plane. By means of the triangle of forces, or otherwise, find the tension in the cord and show that the tension is least when $\theta = 30°$.

4. A wheel of radius 3 feet is rolling, without slipping, on a horizontal plane ; the centre moving with a uniform velocity of 18 feet per second. What is the angular velocity of the wheel ?

What is the velocity of the extremity of the radius which is inclined at 30° to the upward vertical through the centre of the wheel, the angle being measured in the direction of rotation of the wheel ?

5. A 4 lb. weight is drawn up a rough plane, coefficient of friction $\frac{1}{4}$, inclined at 30° to the horizontal by a string, passing over a light smooth pulley at the top of the plane, to which is attached an 8 lb. weight. Find the acceleration.

Calculate the kinetic energy when the system has moved a distance of 8 ft. and show that it is equal to the loss in potential energy minus the work done against friction.

6. A car weighing 2000 pounds starts under the action of a force which decreases *uniformly* from 240 lb. wt. at time $t = 0$ to 80 lb. wt. at $t = 20$ seconds when it remains constant. The resistances to motion remain constant throughout and are equal to 80 lb. wt. Show that the accelerating force at time t $(0 \leq t \leq 20)$ is $160 - 8t$ lb. wt.

Find (*a*) the velocity at time $t = 20$ sec.; (*b*) the distance travelled at time $t = 20$ sec.; (*c*) the horsepower required when $t = 10$ sec. and also when $t > 20$ secs.

7. A gun fires a projectile with initial velocity u and elevation α. Show that (*a*) the range $r = \frac{u^2}{g} \sin 2\alpha$; (*b*) the greatest height $h = \frac{u^2}{2g} \sin^2\alpha$, neglecting air resistance.

Show that the maximum range which can be obtained with the same initial velocity is $2h + \frac{r^2}{8h}$.

8. A cyclist is travelling round a curve 100 feet radius at 15 miles per hour. At what angle must the track be inclined radially so that there may be no tendency to slip ?

If $\mu = 0{\cdot}2$ what is the maximum speed on a level track of the same radius so that skidding may not occur ? (μ is the coefficient of friction).

9. If the period of a simple harmonic vibration is T seconds and the amplitude a feet, show that the maximum acceleration is $4\pi^2 a/T^2$.

A boy weighing 8 stone standing on a plank oscillates vertically in simple harmonic motion of amplitude 0·5 foot and period 1 second. Find the greatest and least reactions of the plank.