

AN ROINN OIDEACHAIS
(Department of Education).

BRAINNSE AN MHEADHON-OIDEACHAIS
(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1939.

HONOURS.

APPLIED MATHEMATICS.

WEDNESDAY, 21st JUNE.—AFTERNOON 4 TO 6 P.M.

Not more than *six* questions may be answered. All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

1. A body moves in a plane, its position referred to rectangular axes in the plane after t seconds being given by

$$x = a + bt, \quad y = c + dt + et^2,$$

where a, b, c, d, e are constants.

Show that it moves with uniform acceleration and find meanings for a, b, c, d, e .

If $e = -g$, the acceleration of gravity, to what does the motion correspond?

2. Show how the velocity of one body relative to another can be represented graphically.

Two cars are travelling, one at 30 and the other at 40 miles per hour, on roads which cross at right angles. Find the magnitude of their relative velocities. If the slower car passed the crossing 10 seconds before the other, what was the distance (in a straight line) between the cars when they were nearest to one another?

3. A wooden chessboard ABCD 16 inches square is made up of sixty-four equal squares. If three of the two-inch squares are broken off, one from each of three corners, find the distance from the fourth corner of the centre of gravity of the remainder. (Answer in inches, to the nearest $\frac{1}{10}$ inch.)

4. A rhombus ABCD is formed by four equal uniform rods loosely jointed together and the system is kept from collapsing by a light inelastic string joining A to C. The system is suspended from the joint A. Find the tension in the string and show that the reaction at B varies as the tangent of the angle which the rods make with the vertical.

5. Define "kinetic energy" and "potential energy".

Show that at all stages of the motion of a body sliding down a smooth plane the sum of the kinetic and potential energies is constant.

When two bodies impinge show that some kinetic energy is lost (or transformed) after impact. [You need consider only the case of inelastic bodies that collide directly.]

6. A rough plane is inclined at an angle α to the horizontal, the coefficient of friction being μ . A mass of m lb. is supported on the plane by a rope which makes an angle θ with the line of greatest slope of the plane. Find expressions for the tension of the rope which (i) will just keep the mass from sliding down, and (ii) will just drag the mass up.

Find the tensions when $m=65$, $\tan \alpha = \frac{5}{12}$, $\tan \theta = \frac{3}{4}$ and $\mu = \frac{1}{3}$.

7. A shell is fired from a point in a horizontal plane in the direction of an object which is let fall at the same instant. Show that the shell will strike the object.

If the masses of the shell and object are equal and if the collision takes place when the shell is at the highest point of its path and assuming that they coalesce there, show that they will strike the plane together at a point which is three-quarters as far from the point of projection as the point which the shell alone would have reached if the object had not been dropped.

8. If a particle is moving in a circle of radius r with velocity v , what is the acceleration directed towards the centre?

Consider the motion of a particle which slides from rest from the highest point down the outside of a smooth sphere and show that it will fall off when it has descended a vertical distance equal to one-third the radius of the sphere.

9. A particle at G, the centre of gravity of a triangle ABC, is acted on by forces represented by GA, GB, GC. Prove that the forces are in equilibrium.

P is a point in the plane of the triangle and forces acting at P are represented by PA, PB, PC. If the magnitude of their resultant is constant, prove that the locus of P is the circle, centre G and radius GP.

10. Two masses of 1 gram and 9 grams respectively rest on a smooth horizontal table and are connected with a light inextensible string which is 2 feet long. The lighter mass is at the edge of the table and the other is a foot from the lighter in a direction at right angles to the edge. The lighter mass falls off the table. Find the common velocity of the masses when the heavier is jerked into motion and find the time it will take to reach the edge of the table.