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LEAVING CERTIFICATE EXAMINATION, 1931.

HONOURS.
APPLIED MATHEMATICS.

WEDNESDAY, 17th JUNE.—AFTERNOON, 4 TO 6 P.M.

Not more than *six* questions may be answered. All questions are of equal value.

Mathematical Tables may be obtained from the Superintendent.

1. The motion of a particle along a straight line is given in the following table :—

Time in seconds	0	·1	·2	·3	·4	·5	·6
Distance in cm.	0	13·5	24·3	29·6	27·5	21·2	9·3
Time in seconds	·7	·8	·90	1·0	1·1	1·2	1·3
Distance in cm.	−4·7	−17·6	−26·7	−30·0	−26·7	−17·6	−4·7

Obtain approximately its velocity at intervals of 0·2 second from 0·2 second to 1·1 seconds, and plot V against T. Hence determine approximately when (1) the velocity of the particle is zero and (2) its acceleration is zero.

Or :

If a particle moves according to the law $x = k \sin t$, explain how you can obtain, by calculus, expressions for its speed, and its acceleration at any instant. When, during the motion, are these magnitudes zero ? How is this kind of motion usually described ?

2. A train acquires a speed of 45 miles an hour in 5 minutes. If the carriage wheels are 4 feet in diameter, what is their angular velocity at this speed ? What was their average angular acceleration ?

Show, on a diagram, the magnitude and direction of the components of the acceleration of the highest point of the wheel during the accelerated motion of the train, and the acceleration of the same point when the train is moving with uniform speed.

3. A stone is projected with velocity v and elevation θ from a point O in a horizontal plane, so as to hit a mark P at a horizontal distance h from O, and at a height k above the plane. Show that v, h, k and θ are connected by the relation :

$$h \sin \theta \cos \theta - k \cos^2 \theta = gh^2/2v^2.$$

Derive the condition that $h^2/2v^2$ should be a maximum when h and k are constants, and show that it is satisfied by $\theta = \pi/4 + \alpha/2$ where α is the elevation of P from O.

4. In an Attwood's machine, a mass of 2 pounds is attached to each end of the cord. An additional mass of 0·6 pound is placed on one side and is found to produce a velocity of 4·72 feet per second at the end of a descent from rest of 4 feet. Compare this result with that given by the simple theory. Express in ft-lb. the kinetic energy of the masses and the work done by the weights, and account for the difference between these quantities.

5. Two uniform rods AB and BC weighing 150 grams each, and of lengths 30 cm. and 40 cm. respectively are freely jointed together at B. They are maintained in a horizontal straight line by three vertical strings, one attached to a point D in AB, 6cm. from A, and the others at B and C. Find the tensions in the strings.

6. A uniform square lamina, ABCD of 9 in. side, is divided into two parts by a line joining A to a point E in DC where DE = 3 in. State the distances of the centres of gravity of the triangles ABE and BCE from the sides AB and BC. Find the distances of the centre of gravity of ABCE from AB and BC.

7. A uniform rod, 106 cm. long and weighing 150 grams, is suspended from a fixed point by strings 90 cm. and 56 cm. long attached to the ends of the rod. Find the tension in each string.

8. Two particles m_1 and m_2 moving along the axes OX and OY respectively towards O with velocities u_1 and u_2 collide at O. What are the components of the velocity of the centre of gravity before the collision? Why do they remain unaltered by the collision? Show that the kinetic energy of the particles is equal to the kinetic energy of a mass m_1+m_2 moving with the velocity of the centre of gravity and a mass $\dfrac{m_1 m_2}{m_1+m_2}$ moving with the relative velocity of either particle with respect to the other.

9. Define the term Power. A motor-car weighing 32 cwt. is travelling at a uniform speed of 27 miles per hour on a level road. On reaching a hill which descends with a uniform gradient of 1 in 25, it is allowed to run free, and the speed is observed to be the same as before. Calculate the resistance of the road and the horsepower expended on the level.

10. A pendulum bob at the end of a string 60 inches long, describes a horizontal circle with the string making an angle of 30 degrees with the vertical. Find its angular velocity and the time of describing the circle. State clearly the principles applied in the solution of this problem.