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(Department of Education).

BRAINSE AN MHEADHON-OIDEACHAIS

(Secondary Education Branch).

LEAVING CERTIFICATE EXAMINATION, 1925

HONOURS.

APPLIED MATHEMATICS.

WEDNESDAY, 24th JUNE—AFTERNOON, 4 *to* 6 P.M.

Six questions may be answered, and the marks will be awarded on the first six answers left uncancelled.

Tables of measures and constants, and of logarithms may be obtained from the Superintendent.

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1. What is meant by 'work,' 'power,' 'one-horse power'?

A locomotive of w tons weight draws a train of W tons at v miles per hour, the resistance to motion being R lbs. per ton. At what horse-power is it working when moving

(a) on the level,

(b) up an incline of 1 in n,

(c) down an incline of 1 in n?

Evaluate when

$w = 8$, $W = 142$, $v = 40$, $R = 10$, $n = 250$.

2. A body moves in a plane, its position referred to rectangular axes in the plane after t seconds being given by

$$x = a + bt, \quad y = c + dt + ft^2.$$

Find its velocity and acceleration in magnitude and direction at any time.

A body is projected from the top of a cliff with a velocity of 45 feet per second, in a direction making 35° upward with the horizontal, and outwards towards the sea, which lies 150 feet below. Where will the body strike the sea?

3. Show that the acceleration of a point in its path is the velocity of the corresponding point on the hodograph.

Find the magnitude and direction of the acceleration of a particle moving in a circle with uniform speed, and derive the period of a conical pendulum of length l inclined at an angle α to the vertical.

4. What are the equations that hold for uniformly accelerated linear motion? Prove the corresponding equations for uniformly accelerated angular motion.

A flywheel, whose mass may be supposed concentrated in the rim of mean radius r feet, possesses K ft.-lb. units of energy when its speed is N revolutions per minute. What is its weight? With how much energy does it part in slowing down to n revolutions per minute? If during the interval of slowing down it has made R revolutions, what was its average angular retardation?

Work out your answers for the case

$$R = 10, \quad r = 3, \quad K = 120{,}000, \quad N = 120, \quad n = 117.$$

5. A motor car of weight W lbs. and axles of length $2a$ feet has its centre of gravity at a height h feet above the ground. Show that when the car turns a corner in a circular arc of radius r feet, with speed v feet per second, the reaction of the outer wheels is altered by an amount $\frac{Wv^2h}{2gra}$ lbs., if the outer wheels carry half the weight of the car when moving in a straight path. What is the greatest speed at which the car could so turn, so that the inner wheels should not lift from the ground?

6. What is meant by velocity ratio, mechanical advantage, and efficiency as applied to machines?

A block and tackle has three pulleys on each block. When a rope passes over a pulley, the tension on one side is ·85 times that on the other. If P is the effort, find the tension in the part of the rope which is fastened to the upper block

(1) when the load is slowly ascending,

(2) when the load is slowly descending.

If the load raised is W, and the weight of the lower block is w, measured in the same units, prove that the efficiency is approximately $= \frac{\cdot 73\,W}{W + w}$.

7. What is simple harmonic motion?

A particle is fastened to the mid-point P of a stretched elastic string, fixed at its ends to two points A and B on a smooth horizontal plane. If P is pulled a small distance x at right angles to AB, prove AP now equals $l\left(1 + \frac{x^2}{2l^2}\right)$ nearly, where $AB = 2l$. Show that the force tending to restore the particle to its original position is $\frac{2Tx}{l}$, approximately, where T is the original tension of the string. Find the time of a small oscillation.

8. Three particles are in motion in a plane. Their masses and positions at time t seconds, referred to rectangular axes in the plane, are given in the following table:—

Mass in lbs.	x feet.	y feet.
3	$3 + 4t + 2t^2$	$7 + 4t + 9t^2$
4	$1 + 2t + 4t^2$	$3 + 3t + 4t^2$
5	$5 + 4t - 2t^2$	$6 - 3t + t^2$

[*Turn over.*

Find expressions for the position, velocity, and acceleration of their centre of gravity at any time.

Verify, for components parallel to the x-axis, that the system of particles moves in that direction as if the whole mass were concentrated at the centre of gravity, and the x-components of the forces acted there.

9. Define an 'impulse,' and show how it is measured.

A jet of water is directed from a circular nozzle 1 inch in diameter with a velocity which would carry it vertically 100 feet, so as to strike horizontally a wall at a height 50 feet above the nozzle: find approximately the pressure on the wall, assuming that the water rebounds with half the velocity of impact.