Zvi Artstein
Mathematics and the Real World. The remarkable role of evolution in the making of mathematics.
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Translated from the Hebrew.

This well-written and entertaining book is aimed at the general public. Technical mathematical material is avoided, although there are occasional more technical notes in a smaller font. The subject matter is the nature of mathematics, and the mathematics of nature, including both the physical and social world. The author's central point is that the human brain (identified with the human mind) is the result of an evolutionary process, and this affects the kind of mathematics we developed, the way we use it, and our ability to cope with it. He rejects the idea that there is such a thing as 'mathematical thinking', if that is taken to mean that mathematicians think in a way that is radically different from other people. This is not to minimize the remarkable achievements made by mathematicians and their impact on human development. Indeed, much of the content of the book celebrates these achievements, showing off the "many facets of mathematics", and invites the reader to learn more about them.

An introductory chapter sets out the basic premise, reviewing the basics of evolution, the mathematical abilities of dumb animals and giving a rapid summary of the historical development of mathematics. The key idea is a division of mathematics into that part (including some arithmetic and geometry) that could have conferred some selective advantage, and the rest. The former comes 'naturally' or 'intuitively' to us, and the latter is 'hard'.

Artstein says that the remaining chapters may be read in any order. Three chapters trace the whole history of mathematics with particular reference to man's developing view of the natural world and with a focus on the major cognitive leaps involved. The remaining chapters deal with the mathematics of randomness from ancient times to Kolmogoroff, human behaviour and economics, computation and computability, foundations and infinity, the modern mathematical research ecosystem, and mathematical education.

The writing is engaging and witty, and the range is vast. Artstein has spent half a lifetime embedded in the mathematical research community, and over the course of the book he explains for a lay reader practically all the topics that come up in coffee-room conversations and colloquium talks. He has a well-judged way of explaining *everything*, without patronizing his reader. His peers will have already met much of the historical and philosophical material, and will note only that the account is largely faithful to received opinion. Even so, though he avoids technicalities and rigorous explanations, experts will find much fascinating information. He draws on evidence gleaned from extensive reading in the scientific and technical literature (not to mention Holy Writ), much of it striking, amusing and instructive. For instance, this reviewer was unaware that Maxwell invented Control Theory when thinking about governors for steam engines, and that Gauss anticipated the Fast Fourier Transform when he computed the orbit of Ceres in 1801-2. In making the case that people can't deal easily with complex logical structures he points out that while we do have idiot savants who can do prodigous arithmetic, we have none who can manage complex logical structures, and that almost everyone misunderstands the sentence: "There is no head injury too minor for you to ignore."

Whether writing about ancient, renaissance or modern astronomy and physics, paradoxes about probability, artificial intelligence, transfinite cardinals, economics, the nature of mathematical research, or elementary education, Artstein keeps coming back to his central idea about the role of evolution in what we do, and can do. This original perspective of his injects a tension into the reading. The reader sees a grand new idea presented as illuminating everything, and is seduced by the thought, but hesitates to yield. Could this really be the answer? We know that many people like mathematics, but we are

acutely aware that most do not. We have bitter experience of the difficulty in teaching mathematics to the masses. Some have said that the first step we must take to succeed as mathematics lecturers is to accept that the students are not the problem: we are the problem, because we do not think like ordinary people. Artstein explicitly rejects this. His thesis is that evolution has left us all with two ways of thinking, one of which operates by default, automatically, when we meet a problem, and he identifies specific characteristics of this way, such as the tendency to ignore improbable events (or risks) and fill in (by assuming or inventing) missing data. He presents evidence to show that we do not easily make rational choices when faced with uncertainty. In his final chapter he reflects on elementary education from this perspective, and his observations deserve to be taken seriously.

It is hard to deny that that the human brain is shaped by an evolutionary process that continued for over a hundred thousand generations. By contrast, the distinctly rational features of our species (or perhaps genus) appeared at most a few thousand generations ago. So it is plausible that human rationality is a graft on a prehuman rational stock. Mathematics as we now think of it, with its emphasis on rigour and precision, is even more recent, less than 150 generations old, and results not from evolution but from something else. Artstein attributes it to the modicum of freedom and leisure enjoyed by some greeks. He gives an interesting cross-classification of types of thinking, into thinking by comparison (involving searching and matching; cannot be learned in the abstract) and creative thinking (based on intuition, feeling, hunches; cannot be learned at all, just encouraged; needs more time in maths than in other disciplines). He quotes Poincaré: "By logic we prove, but by intuition we discover", and points to the helpful role of face-to-face discussion, subconscious activity and sleep in creative thinking. He speculates that our creative thinking utilises different 'brain areas', and may be what distinguishes man from brutes.

Among other controversial views, Artstein rejects the idea that creative thinking declines with age, and the distinction between pure and applied mathematics. His key contention about mathematical education is that understanding cannot be achieved via formal logic. We should distinguish the logical structure of mathematics from the structure for teaching mathematics. He advocates no formal mathematical work before the fourth grade. He says that we do not think conditionally, so probability is difficult and cannot be imparted intuitively. He says there is no chance that we will develop in the student an "intuition for logical operations, mathematical symbols, or other abstract systems without their being rooted in and backed by artihmetic and geometry."

No-one is going to read a book with Mathematics twice in the title unless they have a positive disposition towards the discipline. This book is highly recommended for young people who are that way inclined. Its central idea and its relevance to mathematatical education should be considered seriously by all professionals.

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